AL-FARABI KAZAKH NATIONAL UNIVERSITY

**Faculty of Mathematics and Mechanics**

# Differential Equations and Control Theory Department

**TASKS AND METHODICAL HELP FOR THE PRACTICAL WORKS**

**Architecture of Mathematics**

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**Module 2. Sets**

**Examples of relations and operators**

Week 3

**Examination 1**

Give examples of notions if it exists:

1. Reflexive symmetric, but no transitive relation.

2. Surjection from the set of squares to the set of segments.

**Examination 2**

Give examples of notions if it exists:

1. Operator, which is not surjection and injection ей.

2. Reflexive no symmetric relation of ellipses.

**Examination 3**

Give examples of notions if it exists:

1. Transitive relation of triangles.

2. Surjection from the set of squares to the set of positive numbers.

**Examination 4**

Give examples of notions if it exists:

1. Symmetric no transitive relation.

2. Equivalence of two subsets of set of complex numbers.

**Examination 5**

Give examples of notions if it exists:

1. Bijection between sets of people and real numbers.

2. Reflexive transitive, but no symmetric relation.

**Examination 6**

Give examples of notions if it exists:

1. Injection from the set of triangles to the set of parallelograms.

2. Correspondence between sets of real numbers and lines.

**Examination 7**

Give examples of notions if it exists:

1. Transitive no symmetric relation of complex numbers.

2. Bijection between subset of sets of real numbers and dogs.

**Examination 8**

Give examples of notions if it exists:

1. Reflexive transitive relation of squares.

2. Injection from the set of real numbers to the set of segments.

**Examination 9**

Give examples of notions if it exists:

1. Surjection between the sets of triangles and negative numbers.

2. Reflexive transitive relation of segments.

**Examination 10**

Give examples of notions if it exists:

1. Reflexive symmetric, but no transitive relation between people.

2. Injection from the set of sides of a trapezium to the set of its vertexes.

**Examination 11**

Give examples of notions if it exists:

1. Relation between people, which is not reflexive.

2. Bijection between sets of real and rational numbers.

**Examination 12**

Give examples of notions if it exists:

1. Reflexive symmetric relation of lines.

2. Surjection from the set of numbers to the set of balls.

**Examination 13**

Give examples of notions if it exists:

1. Reflexive no symmetric relation of parabolas.

2. Bijection between sets of sides of a square and cube.

**Examination 14**

Give examples of notions if it exists:

1. Injection from the set of vertexes of cube to the set of natural numbers.

2. Transitive no symmetric relation of squares.

**Examination 15**

Give examples of notions if it exists:

1. Injection from the set of натуральных numbers to the set of вершин куба.

2. Correspondence between sets of sides and vertexes of a rectangle.

**Examination 16**

Give examples of notions if it exists:

1. Symmetric no reflexive relation of curves.

2. Injection from the set of natural numbers to the set of real numbers.

**Examination 17**

Give examples of notions if it exists:

1. Transitive no reflexive relation людей.

2. Surjection from the set of natural numbers to the set of real numbers.

**Examination 18**

Give examples of notions if it exists:

1. Reflexive symmetric relation of triangles.

2. Bijection between sets of rational and real numbers.

**Module 3. Numbers**

**Operators on sets of numbers. Operations on sets of numbers.**

Week 5

**Examination 1**

1. Give example of the object if it exists: injection .

2. Prove the commutativity of the multiplication on the set Z, if this property is true on the set N.

**Examination 2**

1. Give example of the object if it exists: surjection .

2. Prove the associativity of the addition on the set Z, if this property is true on the set N.

**Examination 3**

1. Give example of the object if it exists: surjection .

2. Prove the commutativity of the addition on the set Q, if this property is true on the set Z.

**Examination 4**

1. Give example of the object if it exists: injection .

2. Prove the associativity of the multiplication on the set Q, if this property is true on the set Z.

**Examination 5**

1. Give example of the object if it exists: surjection .

2. Prove the commutativity of the addition on the set R, if this property is true on the set Q.

**Examination 6**

1. Give example of the object if it exists: surjection .

2. Prove the commutativity of the multiplication on the set Z, if this property is true on the set N.

**Examination 7**

1. Give example of the object if it exists: surjection .

2. Prove the associativity addition on the set Z, if this property is true on the set N.

**Examination 8**

1. Give example of the object if it exists: injection .

2. Prove the associativity addition on the set C, if this property is true on the set R.

**Examination 9**

1. Give example of the object if it exists: injection .

2. Prove the commutativity of the addition on the set C, if this property is true on the set R.

**Examination 10**

1. Give example of the object if it exists: injection .

2. Prove the commutativity of the addition on the set Q, if this property is true on the set Z.

**Examination 11**

1. Give example of the object if it exists: surjection .

2. Prove the associativity addition on the set Z, if this property is true on the set N.

**Examination 12**

1. Give example of the object if it exists: surjection .

2. Prove the associativity addition on the set C, if this property is true on the set R.

**Examination 13**

1. Give example of the object if it exists: injection .

2. Prove the commutativity of the addition on the set Q, if this property is true on the set Z.

**Examination 14**

1. Give example of the object if it exists: surjection .

2. Prove the associativity addition on the set Z, if this property is true on the set N.

**Examination 15**

1. Give example of the object if it exists: surjection .

2. Prove the associativity addition on the set C, if this property is true on the set R.

**Examination 16**

1. Give example of the object if it exists: surjection .

2. Prove the commutativity of the multiplication on the set Z, if this property is true on the set N.

**Examination 17**

1. Give example of the object if it exists: injection .

2. Prove the associativity addition on the set Z, if this property is true on the set N.

**Examination 18**

1. Give example of the object if it exists: injection .

2. Prove the associativity of the addition on the set Z, if this property is true on the set N.

**Module 4. Ordered objects**

**Examples and properties of ordered sets**

Week 7.

**Examination 1**

Give example of objects:

1. The set with largest element without smallest element.

2. The quasi ordered set of people.

**Examination 2**

Give example of objects:

1. The linear ordered non numerical set.

2. The set, which is lower bounded but not upper bounded.

**Examination 3**

Give example of objects:

1. The set with two maximal elements.

2. An ordered set of circles.

**Examination 4**

Give example of objects:

1. The set with two minimal elements.

2. The quasi ordered set of triangles.

**Examination 5**

Give example of objects:

1. The linear ordered set, which is not completely ordered.
2. The set with three minimal elements.

**Examination 6**

Give example of objects:

1. The bounded set of circles.

2. The monotone operator on non numerical sets.

**Examination 7**

Give example of objects:

1. The set, which is upper bounded but not lower bounded.

2. Anti monotone operator on non numerical sets.

**Examination 8**

Give example of objects:

1. Unbounded set of circles.

2. Operator, which is monotone and anti monotone.

**Examination 9**

Give example of objects:

1. The set with a smallest element without a largest element.
2. Operator, which is not monotone and anti monotone.

**Examination 10**

Give example of objects:

1. Interval on the ordered set of balls.

2. The quasi ordered set of elephants.

**Examination 11**

Give example of objects:

1. The isomorphism of orders on non numerical sets.

2. Unbounded set of triangles.

**Examination 12**

Give example of objects:

1. The set with two largest elements.

2. The quasi ordered set of vectors.

**Examination 13**

Give example of objects:

1. The set with two maximal elements and one minimal element.

2. The quasi ordered set of ellipses.

**Examination 14**

Give example of objects:

1. The set without largest and smallest elements.

2. The quasi ordered set of balls.

**Examination 15**

Give example of objects:

1. The set with two minimal elements and one maximal element.

2. The isomorphism of order on non numerical sets.

**Examination 16**

Give example of objects:

1. Bounded set of circles.

2. The set with three maximal elements.

**Module 5. Algebraic objects**

**Examples and properties of algebraic objects**

Week 9

**Examination 1**

1. Determine a semigroup on the set continuous functions.

2. Determine a linear functional on the set of integrable functions.

**Examination 2**

1. Determine a monoid on the set of two elements.

2. Determine a linear space on the set of trigonometric polynomials.

**Examination 3**

1. Determine a groupoid but not monoid on the set of three elements.

2. Determine a linear space, which is isomorphs to .

**Examination 4**

1. Determine a but not group on the set negative numbers.

2. Determine a linear space on the set of polynomials.

**Examination 5**

1. Determine a commutative monoid on the set of polynomials.

2. Determine a linear operator from the plane to the set of continuous functions.

**Examination 6**

1. Determine a group on the set of circles.

2. Determine a linear operator from the set of numbers to the set of continuous functions.

**Examination 7**

1. Determine non commutative groupoid on the set of two elements.

2. Determine a linear operator from the set of continuous functions to the plane.

**Examination 8**

1. Determine a semigroup on the set of even numbers.

2. Determine a linear functional on the set differentiable functions.

**Examination 9**

 1. Determine a monoid but not a group on the set of three elements.

2. Determine a subspace of the space of complex numbers.

**Examination 10**

1. Determine a groupoid on the set camels.

2. Determine a linear operator from the plan to the space.

**Examination 11**

1. Determine a non commutative groupoid on the set of sides of triangles.

2. Determine a non convex set of complex numbers.

**Examination 12**

1. Determine a group on the set множестве piecewise constant functions.

2. Determine a convex set of complex numbers.

**Examination 13**

1. Determine a non commutative groupoid on the set of tops of triangles.

2. Determine a one-dimensional subspace of continuous functions.

**Examination 14**

1. Determine a non commutative groupoid on the set of two elements and its dual groupoids.

2. Determine a linear operator from the space to the plane.

**Examination 15**

1. Determine a non commutative groupoid on the set of continuous functions.

2. Determine a finite dimensional subspace of the space of continuous functions.

**Examination 16**

1. Determine commutative groupoid on the set of two elements.

2. Determine a convex set of continuous functions.

**Module 6. Topological objects**

**Examples and properties of topological objects**

Week 11

**Examination 1**

1. Determine a weakest topology on the set of three points. Find neighbourhoods of all points.

2. Determine a disconnected set with connected boundary on the plane.

**Examination 2**

1. Determine a strongest topology on the set of three points. Find neighbourhoods of all points.

2. Determine a set with open boundary on the line.

**Examination 3**

1. Determine a connected topology on the set of three points.

2. Determine two sets on the plane, which are equivalent with respect to the set theory but not to topology.

**Examination 4**

1. Determine a non connected topology on the set of three points.

2. Is it possible a set of the line be equivalent to an open set with respect to the set theory? Explain it if it is impossible. Give an example if it is possible.

**Examination 5**

1. Determine two equivalent sets on the line, one of them is closed and second is not closed.

2. Determine a metric on the set of three elements.

**Examination 6**

1. Determine an inseparable topology on the set of three points.

2. Determine a set on the line with three points of boundary.

**Examination 7**

1. Determine a topology on the set of three points. Find all open, closed and other sets.

2. Determine a two connected but not homeomorphic sets.

**Examination 8**

1. Determine a topology on the set of three points, which is not discrete and indiscrete.

2. Determine a metric on the set bounded functions.

**Examination 9**

 1. Determine a non connected set on the plane with non bounded boundary if it is possible. Explain the situation if it is impossible.

2. Is it possible to determine a metric on the set of three elements with unbounded set? Explain it if it is impossible. Give an example if it is possible.

**Examination 10**

1. Determine two topologies on the set of three points. Determine a sequence, which converges in first topology and diverges in the second one.

2. Determine a non connected closed set on the line.

**Examination 11**

1. Determine two closed sets of the line, which are non equivalent.

2. Determine a non connected bounded set of the line. Find its diameter.

**Examination 12**

1. Determine a separable topology on the set of three elements. Determine its converged sequence.

2. Determine a metric on the set of complex numbers. Determine the bounded set there. Find its diameter.

**Examination 13**

1. Determine two equivalent sets of the line, one of them is open and second is not open.

2. Determine a metric on the set of four elements.

**Examination 14**

1. Is the equivalence of sets a topological property? Explain this situation.

2. Determine a metric on the set of elephants.

**Examination 15**

1. Determine a non connected set on the plane with boundary, which contain one point.

2. Determine a metric on the set of circles.

**Examination 16**

1. Determine a separable topology on the set of three points.

2. Determine a non connected open set on the line.

**Module 6. Measurable** **objects**

**Examples and properties of measurable** **topological objects**

Week 13

**Examination 1**

1. Determine a measurable space on the circle, which is algebra.

2. Determine a measure on the set of four points.

**Examination 2**

1. Determine a σ-algebra on the set of four points.

2. Determine Lebesgue measure of a non connected closed set of the plane.

**Examination 3**

1. Determine a ring on the set of four points, which is not σ-algebra.

2. Determine a measure a on the set of circles.

**Examination 4**

1. Determine a measurable space on the set of squares.

2. Determine Lebesgue measure of a semi open interval.

**Examination 5**

1. Determine algebra on the set of sides of squares.

2. Determine a probabilistic measure on the set four points.

**Examination 6**

1. Determine a measurable space on the set of three points, which is not a topological space.

2. Determine a measure on a non connected open set of the plane.

**Examination 7**

1. Determine a measurable space on the set of elephants.

2. Find of Lebesgue measure of a non closed connected set of the line.

**Examination 8**

1. Determine a σ-algebra on the set of five points.

2. Is it possible a discontinuous operator, which saves a measure? Explain it if it is impossible. Give an example if it is possible.

**Examination 9**

 1. Is it possible algebra on the sides of a square, which is not σ-algebra? Explain it if it is impossible. Give an example if it is possible.

2. Determine a measure on the set of line with boundary, which has three points.

**Examination 10**

1. Determine a σ-algebra on the boundary of a square.

2. Determine Lebesgue measure of on the boundary of a semi open interval.

**Examination 11**

1. Determine σ-algebra on the set of tops of triangle.

2. Determine Lebesgue measure Лебега of an open unbounded interval on the line.

**Examination 12**

1. Determine algebra but not σ-algebra on the subset of the line with boundary, which contains one point, if it is possible. Explain it if it is impossible.

2. Determine a discontinuous operator on the plane, which save Lebesgue measure.

**Examination 13**

1. Is it possible to determine a ring but not algebra on the set with three points? Explain it if it is impossible. Give an example if it is possible.

2. Determine a measure on the set with four points.

**Examination 14**

1. Is it possible to determine algebra but not σ-algebra on the set with three points? Explain it if it is impossible. Give an example if it is possible.

2. Determine a continuous operator on the line, which does not save Lebesgue measure.

**Examination 15**

1. Determine algebra on disconnected set on the line with boundary, which has a two points.

2. Determine Lebesgue measure on the closed set of the line.

**Examination 16**

1. Determine a measurable space on the set of two points, which is not a topological space.

2. Determine a measure on the set of line with boundary, which has one point.